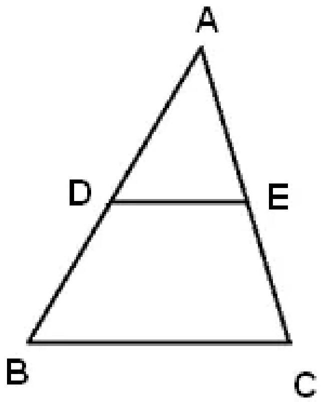
Chapter 16. Similarity

Ex 16.1

Answer 1.



(i) In $\triangle ADE$ and $\triangle ABC$

$\angle D = \angle B$ and $\angle C = \angle E$ $\quad$ (DE||BC)

$\Rightarrow \triangle ADE \sim \triangle ABC$

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{4}{x-4} = \frac{8}{3x-19}$

$\Rightarrow 4x(3x-19) = 8x(x-4)$

$\Rightarrow 12x - 76 = 8x - 32$

$\Rightarrow 4x = 44$

$\Rightarrow x = 11$

(ii) In ΔADE and ΔABC

$\angle D = \angle B$ and $\angle C = \angle E$ (DE||BC)

$\Rightarrow \Delta ADE \sim \Delta ABC$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{4}{5} = \frac{AE}{2.5}$$

$$\Rightarrow AE = \frac{4 \times 2.5}{5}$$

$$\Rightarrow AE = 2cm$$

(iii) In ΔADE and ΔABC

$\angle D = \angle B$ and $\angle C = \angle E$ (DE||BC)

$\Rightarrow \Delta ADE \sim \Delta ABC$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$$

$$\Rightarrow (4x-3) \times (5x-3) = (8x-7) \times (3x-1)$$

$$\Rightarrow 20x^2 - 15x - 12x + 9 = 24x^2 - 21x - 8x + 7$$

$$\Rightarrow 20x^2 - 27x + 9 = 24x^2 - 29x + 7$$

$$\Rightarrow 4x^2 - 2x - 2 = 0$$

$$\Rightarrow x(x-1) + \frac{1}{2}(x-1) = 0$$

$$\Rightarrow (x + \frac{1}{2}) = 0; x - 1 = 0$$

$$\Rightarrow x = -\frac{1}{2}; x = 1$$

$$\therefore x = 1$$

(iv) In ΔADE and ΔABC

$\angle D = \angle B$ and $\angle C = \angle E$ (DE||BC)

$\Rightarrow \Delta ADE \sim \Delta ABC$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

$DB = AB - AD = 12 - 8 = 4$

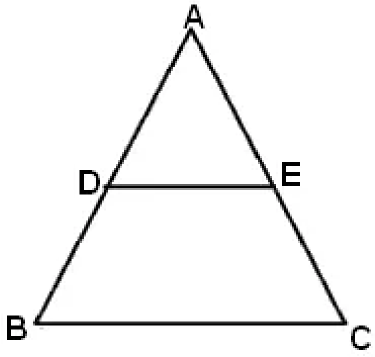
$$\Rightarrow \frac{8}{4} = \frac{12}{EC}$$

$$\Rightarrow 8 \times EC = 12 \times 4$$

$$\Rightarrow EC = \frac{12 \times 4}{8}$$

$$\Rightarrow EC = 6cm$$

Answer 2.



(i) AB=5.6 cm, AD=1.4 cm, AC=7.2 cm and AE=1.8 cm

$$\frac{AD}{AB}=\frac{1.4}{5.6}=\frac{7}{28}=\frac{1}{4}$$
$$\frac{AE}{AC}=\frac{1.8}{7.2}=\frac{2}{8}=\frac{1}{4}$$
$$\Rightarrow \frac{AD}{AB}=\frac{AE}{AC}$$
$$\therefore \Delta ADE \sim \Delta ABC$$
$$\Rightarrow \angle D=\angle B; \angle E=\angle C$$

But these are corresponding angles

Hence, DE||BC

(ii) AB=10.8 cm, BD=4.5 cm, AC=4.8 cm and AE=2.8 cm

AD=AB-BD = 10.8 – 4.5 = 6.3 cm

$$\frac{AD}{AB}=\frac{6.3}{10.8}=\frac{21}{36}=\frac{7}{12}$$
$$\frac{AE}{AC}=\frac{2.8}{4.8}=\frac{14}{24}=\frac{7}{12}$$
$$\Rightarrow \frac{AD}{AB}=\frac{AE}{AC}$$
$$\therefore \Delta ADE \sim \Delta ABC$$
$$\Rightarrow \angle D=\angle B; \angle E=\angle C$$

But these are corresponding angles

Hence, DE||BC

(iii) AD=5.7 cm, BD=9.5 cm, AE=3.3 cm and EC=5.5 cm

$$\frac{AD}{BD}=\frac{5.7}{9.5}=0.6$$
$$\frac{AE}{EC}=\frac{3.3}{5.5}=\frac{3}{5}=0.6$$
$$\Rightarrow \frac{AD}{BD}=\frac{AE}{EC}$$
$$\therefore \Delta ADE \sim \Delta ABC$$
$$\Rightarrow \angle D=\angle B; \angle E=\angle C$$

But these are corresponding angles

Hence, DE||BC

Answer 3.

(i) Since PQ||BC

$$\frac{AP}{PB} = \frac{AQ}{QC}$$

$$\Rightarrow \frac{AP}{AB - AP} = \frac{AQ}{QC}$$

$$\Rightarrow \frac{2}{5} = \frac{AQ}{10}$$

$$\Rightarrow AQ = \frac{2 \times 10}{5}$$

$$\Rightarrow AQ = 4$$

(ii) Since PQ||BC

$$\frac{AP}{AB} = \frac{PQ}{BC}$$

$$\Rightarrow \frac{2}{7} = \frac{PQ}{21}$$

$$\Rightarrow PQ = \frac{2 \times 21}{7}$$

$$\Rightarrow PQ = 6$$

Answer 4.

(i) Since DE||BC

$$\frac{DE}{BC} = \frac{AD}{AB}$$

$$\Rightarrow \frac{3}{8} = \frac{AD}{AB} \Rightarrow \frac{AD}{AB} = \frac{3}{8}$$

Since DB=AB-AD

$$\Rightarrow DB = 8 - 3 = 5$$

Therefore, AD:DB = 3:5

(ii) DE:BC=3:8

Since DE||BC

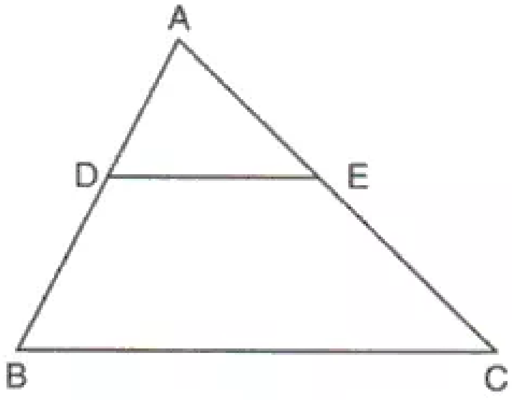
$$\frac{DE}{BC} = \frac{AE}{AC}$$

$$\Rightarrow \frac{3}{8} = \frac{AE}{16}$$

$$\Rightarrow AE = \frac{3 \times 16}{8}$$

$$\Rightarrow AE = 6$$

Answer 5.



Considering DE || BC

(i) $\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{AE}{EC} = \frac{AD}{DB}$

$\Rightarrow \frac{AE}{EC} = \frac{5}{7}$

(ii) $\frac{AD}{DB} = \frac{5}{7}$

$\because AB = AD + DB$

$\Rightarrow AB = 5 + 7 = 12$

$\therefore \frac{AD}{AB} = \frac{5}{12}$

(iii) $\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{AE}{EC} = \frac{AD}{DB}$

$\Rightarrow \frac{AE}{EC} = \frac{5}{7}$

$\because AC = AE + EC$

$\Rightarrow AC = 5 + 7 = 12$

$\therefore \frac{AE}{AC} = \frac{5}{12}$

(iv) Since DE||BC

$\frac{AD}{AB} = \frac{DE}{BC}$

$\Rightarrow \frac{5}{12} = \frac{2.5}{BC}$

$\Rightarrow BC = \frac{2.5 \times 12}{5}$

$\Rightarrow BC = 6cm$

(v) Since DE||BC

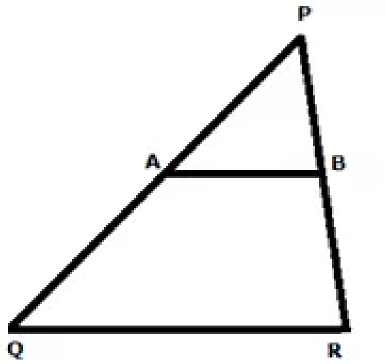
$$\frac{AD}{AB} = \frac{DE}{BC}$$

$$\Rightarrow \frac{5}{12} = \frac{DE}{4.8}$$

$$\Rightarrow DE = \frac{5 \times 4.8}{12}$$

$$\Rightarrow BC = 2cm$$

Answer 6.



AB||QR

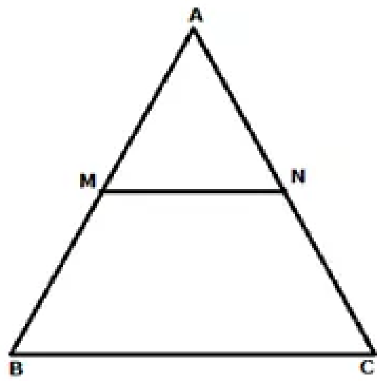
$$\frac{AP}{PQ} = \frac{PB}{PR}$$

$$\Rightarrow \frac{AP}{9} = \frac{4.2}{6}$$

$$\Rightarrow AP = \frac{4.2 \times 9}{6}$$

$$\Rightarrow AP = 6.3cm$$

Answer 7.



(i) $\frac{AM}{AB} = \frac{5}{7}$

$\because AB = 3.5cm$

$$\therefore AM = \frac{5 \times AB}{7}$$

$$\Rightarrow AM = \frac{5 \times 3.5}{7}$$

$$\Rightarrow AM = 2.5cm$$

(ii) Since MN||BC and $\frac{AM}{MB} = \frac{AN}{NC}$

$\because AB = 3.5cm; AM = 2.5cm$

$\therefore MB = AB - AM = 3.5 - 2.5 = 1cm$

$\Rightarrow \frac{AM}{MB} = \frac{AN}{NC}$

$\Rightarrow \frac{2.5}{1} = \frac{AN}{2}$

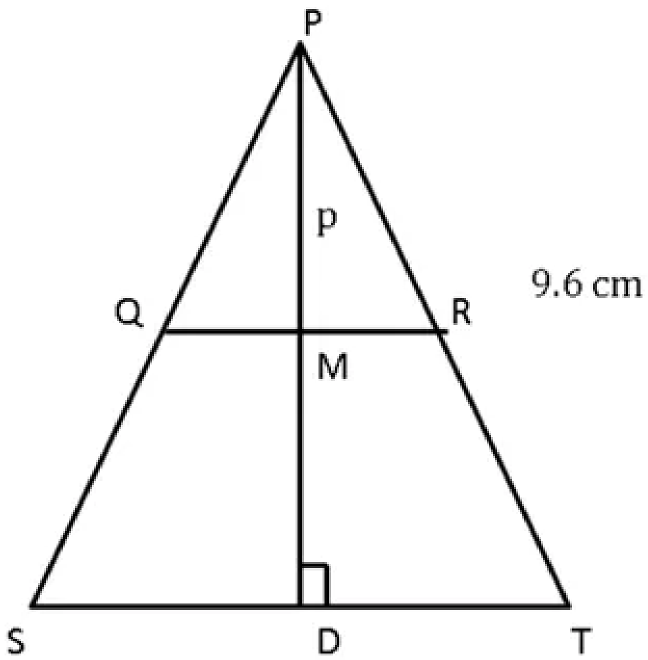
$\Rightarrow AN = \frac{2.5 \times 2}{1} = 5cm$

Now,

$AC = AN + NC$

$\Rightarrow AC = 5 + 2 = 7cm$

Answer 8.



Since QR is parallel to ST,

By Basic Theorem of Proportionality,

$\frac{PQ}{PS} = \frac{PR}{PT}$

$\Rightarrow \frac{3}{4} = \frac{PR}{9.6}$

$\Rightarrow PR = \frac{9.6 \times 3}{4} = 7.2 \text{ cm}$

Since QR is parallel to ST,

QM || SD

By Basic Theorem of Proportionality,

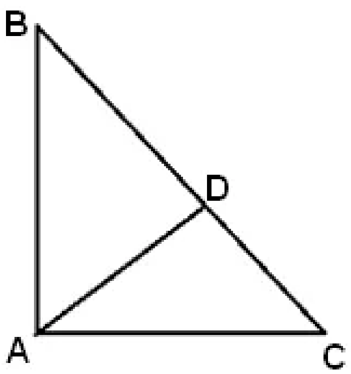
$$\frac{PQ}{PS} = \frac{PM}{PD}$$

$$\Rightarrow \frac{3}{4} = \frac{p}{PD}$$

$$\Rightarrow PD = \frac{4p}{3}$$

So, the length of the perpendicular from P to ST in terms of p is $\frac{4p}{3}$.

Answer 9.



In ΔABC,

Using Pythagoras theorem

$$BC^2 = AB^2 + AC^2$$
$$BC^2 = 8^2 + 6^2$$
$$BC^2 = 64 + 36$$
$$BC = \sqrt{100} = 10 \text{..........(i)}$$

In ΔABD,

Using Pythagoras theorem

$AD^2 = AB^2 - BD^2$

$AD^2 = 8^2 - BD^2 ......(ii)$

In ΔACD,

Using Pythagoras theorem

$AD^2 = AC^2 - CD^2$

$AD^2 = 6^2 - CD^2 ......(iii)$

Equating (ii) and (iii)

$8^2 - BD^2 = 6^2 - CD^2$

$\because CD = BC - BD$

$8^2 - BD^2 = 6^2 - (BC - BD)^2$

$CD = BC - BD$

$BC = 10cm(\text{from}(i))$

$8^2 - BD^2 = 6^2 - (10 - BD)^2$

$8^2 - BD^2 = 6^2 - (100 - 20BD + BD^2)$

$64 - BD^2 = 36 - 100 + 20BD - BD^2$

$64 = -64 + 20BD$

$20BD = 128$

$BD = 6.4cm$

Answer 10.

In ΔPRT and ΔSQT

$\angle PTR = \angle STQ$ (vertically opposite angles)

$\angle RPT = \angle SQT$ (alternate angles $\because PR||SQ$)

$\therefore \Delta PRT \cong \Delta SQT$

$\Rightarrow \frac{RT}{PT} = \frac{ST}{TQ}$

$\Rightarrow \frac{RT}{5} = \frac{9}{6}$

$\Rightarrow RT = \frac{5 \times 9}{6}$

$\Rightarrow RT = 7.5cm$

Also,

$\frac{PT}{PR} = \frac{TQ}{SQ}$

$\Rightarrow \frac{5}{10} = \frac{6}{SQ}$

$\Rightarrow SQ = \frac{6 \times 10}{5}$

$\Rightarrow SQ = 12cm$

Answer 11.

In ΔCGB and ΔAGP

$\angle CGB = \angle AGP$ (vertically opposite angles)

$\angle GAP = \angle GCB$ (AD || BC, therefore alternate angles)

Therefore, $\Delta CGB \sim \Delta AGP$ (AA axiom)

$$\therefore \frac{CG}{GA} = \frac{BC}{AP}$$

$$\Rightarrow \frac{3}{5} = \frac{12}{AP}$$

$$\Rightarrow AP = \frac{5 \times 12}{3}$$

$$\Rightarrow AP = 20cm$$

Answer 12.

(i) In ΔOBQ and ΔOPC

$\angle OQB = \angle OPC = 90°$ (QC and BP are altitudes)

$\angle QOB = \angle POC$ (vertically opposite angles)

Therefore, $\Delta OBQ \sim \Delta OPC$

$$\Rightarrow \frac{PC}{OP} = \frac{QB}{OQ}$$

$$\Rightarrow PC \times OQ = QB \times OP$$

(ii) Since $\Delta OBQ \sim \Delta OPC$

$$\frac{OC}{PO} \times \frac{OC}{PC} = \frac{OB}{QB} \times \frac{OB}{QO}$$

$$\Rightarrow \frac{OC^2}{PC \times PO} = \frac{OB^2}{QB \times QO}$$

$$\Rightarrow \frac{OC^2}{OB^2} = \frac{PC \times PO}{QB \times QO}$$

Answer 13.

In ΔPQS and ΔQTR

$\angle PQS = \angle TQR$ (vertically opposite angles)

$\angle SPQ = \angle QRT$ (alternate angles)

Therefore, $\Delta PQS \sim \Delta QTR$

$$\Rightarrow \frac{PQ}{QS} = \frac{QR}{QT}$$
$$\Rightarrow \frac{PQ}{12} = \frac{15}{10}$$
$$\Rightarrow PQ = \frac{15 \times 12}{10}$$
$$\Rightarrow PQ = 18cm$$

Also,

$$\Rightarrow \frac{QS}{PS} = \frac{QT}{RT}$$
$$\Rightarrow \frac{12}{PS} = \frac{10}{6}$$
$$\Rightarrow PS = \frac{6 \times 12}{10}$$
$$\Rightarrow PQ = 7.2cm$$

Answer 14.

In ΔPQA and ΔDQC

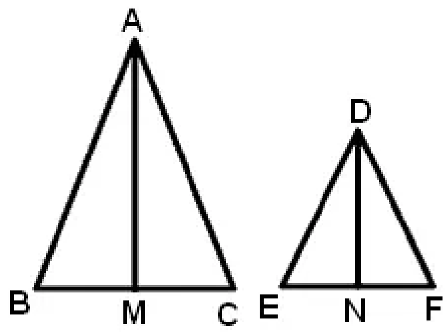
$\angle PQA = \angle DQC$ (vertically opposite angles)

$\angle APQ = \angle QDC$ (alternate angles since AB||DC)

Therefore, $\Delta PQA \sim \Delta DQC$

$$\therefore \frac{CQ}{QD} = \frac{QA}{PQ}$$
$$\Rightarrow CQ \times PQ = QA \times QD$$

Answer 15.



Since $\Delta ABC \sim \Delta DEF$

$\angle B = \angle E$

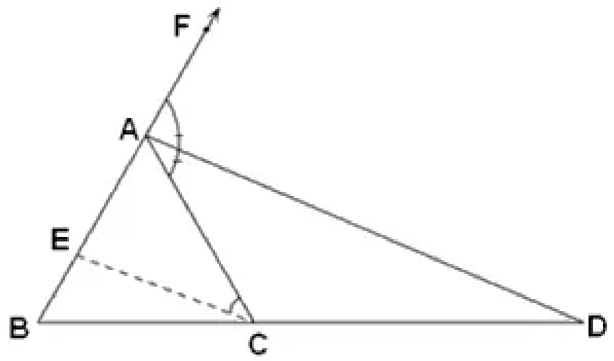
$\angle AMB = \angle DNE$ (Both are right angles)

Therefore, $\Delta ANB \sim \Delta DNE$

$\therefore \frac{AM}{DN} = \frac{AB}{DE}$

$\Rightarrow AM : DN = AB : DE$

Answer 16.



In ΔABC, $CE || AD$

$\therefore \frac{BD}{CD} = \frac{AB}{AE}$ .....(i)

(By Basic Pr oportionality theorem)

AD is the bisector of $\angle CAF$

$\angle FAD = \angle CAD$......(ii)

Since $CE || AD$

Therefore,

$\angle ACE = \angle CAD$......(iii) (alternate angles)

$\angle AEC = \angle FAD$......(iv) (corresponding angles)

From (ii), (iii) and (iv)

$\angle AEC = \angle ACE$

In ΔAEC,

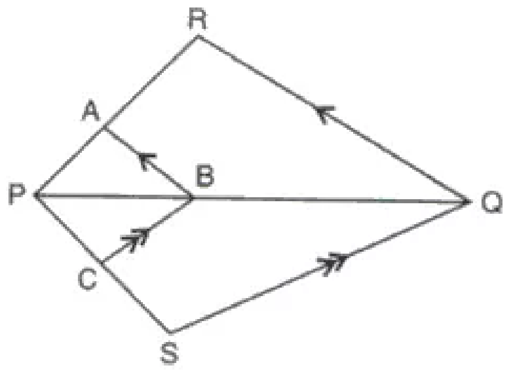
$\angle AEC = \angle ACE$

$\Rightarrow AC = AE$ ....(v) (Equal angles have equal sides opposite to them)

From (i) and (v)

$$\frac{BD}{CD} = \frac{AB}{AC}$$

Answer 17.



In ΔPQR, $AB || RQ$

$$\therefore \frac{PA}{PR} = \frac{PB}{PQ} \text{.......(i)(By Basic Proportionality theorem)}$$

In ΔPQS, $BC || SQ$

$$\therefore \frac{PC}{PS} = \frac{PB}{PQ} \text{.......(ii)(By Basic Proportionality theorem)}$$

From (i) and (ii)

$$\frac{PC}{PS} = \frac{PA}{PR}$$

Answer 18.

In ΔABC, $DE || AC$

$$\therefore \frac{BE}{EC} = \frac{BD}{DA} \text{.......(i)(By Basic Proportionality theorem)}$$

In ΔABP, $CD || AP$

$$\therefore \frac{BC}{CP} = \frac{BD}{DA} \text{.......(ii)(By Basic Proportionality theorem)}$$

From (i) and (ii)

$$\frac{BE}{EC} = \frac{BC}{CP}$$

Answer 19.

In ΔABD and ΔAPQ,

$\angle BDA = \angle PQA = 90^\circ$

$\angle A = \angle A$

Therefore, $\Delta ABD \sim \Delta APQ$ (AA axiom)

And hence, $\frac{AB}{AP} = \frac{BD}{PQ}$

Answer 20.

In ΔPMS and ΔMQN

$\angle PMS = \angle NMQ$ (vertically opposite angles)

$\angle SPM = \angle MQN$ (alternate angles, since PS||QN)

Therefore, $\Delta PMS \sim \Delta MQN$

$\therefore \frac{SP}{PM} = \frac{MQ}{QN}$ ....(i)

In ΔPMS and ΔMRS

$\angle PMS = \angle MSR$ (alternate angles, since PM||SR)

$SM = SM$

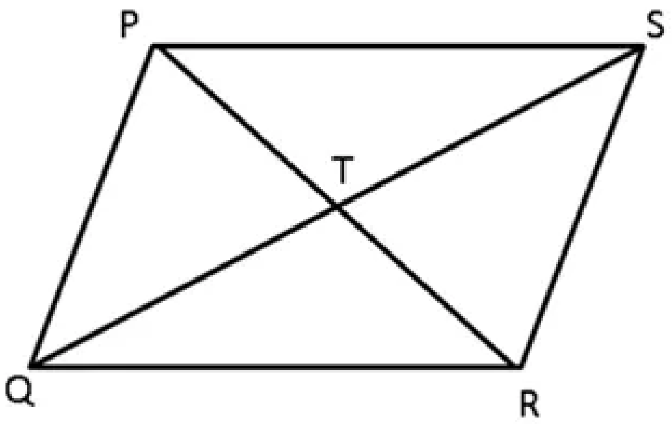
Therefore, $\Delta PMS \sim \Delta MRS$

$\therefore \frac{SP}{PM} = \frac{MR}{SR}$ ....(ii)

From (i) and (ii)

$\therefore \frac{SP}{PM} = \frac{MQ}{QN} = \frac{MR}{SR}$

Answer 21A.



Consider ΔPTQ and ΔRTS,

$\frac{PT}{TR} = \frac{QT}{TS} = \frac{1}{2}$ (Given)

$\angle PTQ = \angle RTS$ (Vertically Opposite angles)

$\Rightarrow \Delta PTQ \sim \Delta RTS$ (SAS criterion for Similarity)

Answer 21B.

Consider ΔPTQ and ΔRTS,

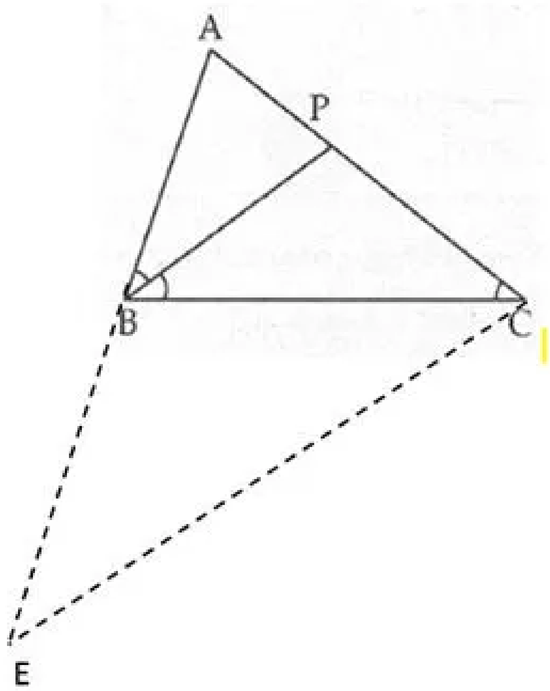
$\frac{PT}{TR} = \frac{QT}{TS} = \frac{1}{2}$ (Given)

$\angle PTQ = \angle RTS$ (Vertically Opposite angles)

$\Rightarrow \Delta PTQ \sim \Delta RTS$ (SAS criterion for Similarity)

$\Rightarrow \frac{TP}{TQ} = \frac{TR}{TS}$ (Re arranging the terms)

Answer 22A.



a Construction : Draw CE || BP and produce AB to E.

Proof : BP || EC

$\angle PBC = \angle BCE$ (Alternate angles)

$\angle ABP = \angle AEC$ (Corresponding angles)

Also, $\angle ABP = \angle PBC$

$\Rightarrow \angle BCE = \angle BEC$

So, BE = BC

In ΔAEC,

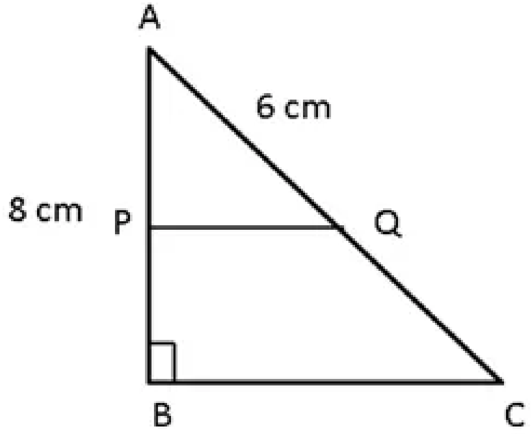
$$\frac{AP}{PC} = \frac{AB}{BE}$$

$$\Rightarrow \frac{AP}{PC} = \frac{AB}{BC}$$

$$\Rightarrow BC \times AP = PC \times AB$$

b. Note : It is not possible to prove this part due to inadequate data.

Answer 23.



In right - angled ΔABC,

$PQ \parallel BC$

$\Rightarrow \frac{PA}{AB} = \frac{QA}{AC}$

$\Rightarrow \frac{1}{3} = \frac{6}{AC}$

$\Rightarrow AC = 18 \text{ cm}$

By Pythagoras Theorem,

$BC^2 = AC^2 - AB^2$

$\Rightarrow BC^2 = 18^2 - 8^2$

$\Rightarrow BC^2 = 324 - 64$

$\Rightarrow BC = 16.12 \text{ cm}$

Ex 16.2

Answer 1.

$\Delta ABC \sim \Delta PRQ$

$A \leftrightarrow P, B \leftrightarrow R, C \leftrightarrow Q$

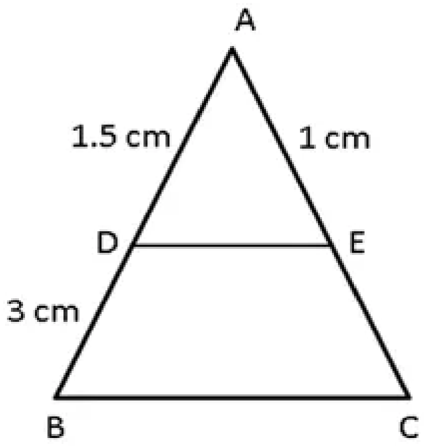
$\angle A \sim \angle P$

$\angle B \sim \angle R$

$\angle C \sim \angle Q$

$AB \sim PR, BC \sim RQ, AC \sim PQ$

Answer 2.



DE || BC

$$\frac{AD}{AB} = \frac{AE}{AC}$$

$$\Rightarrow \frac{1.5}{4.5} = \frac{1}{AC}$$

$$\Rightarrow AC = 3 \text{ cm}$$

Answer 3.

Since the two triangles are similar,
so the ratio of the corresponding sides are equal.
Let x and y be the sides of the triangle,
where y is the longest side.

$$\frac{3}{5} = \frac{4.5}{x} \Rightarrow x = 7.5 \text{ cm}$$

$$\frac{5}{6} = \frac{7.5}{y} \Rightarrow y = 9 \text{ cm}$$

So, the sides of the triangles are 4.5 cm, 7.5 cm and 9 cm.

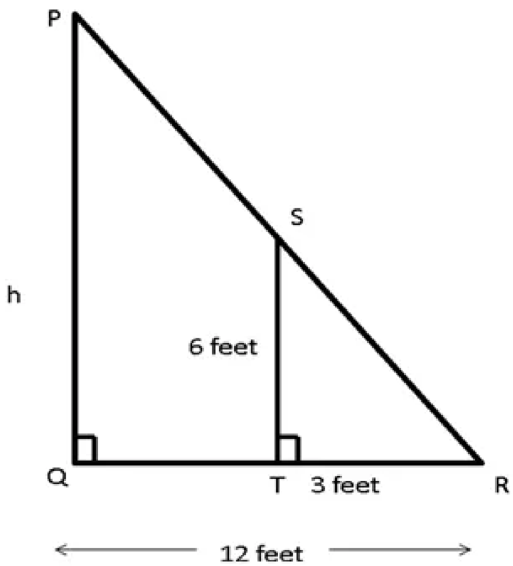
Answer 4.

We know that,
for two similar triangles, ratio of the corresponding sides is equal to ratio of the perimeters of the triangles.

$\Rightarrow$ Ratio of the corresponding sides $= \frac{8}{16} = \frac{1}{2}$

that is, ratio of the corresponding sides is 1 : 2.

Answer 5.



Harmeet and the pole will be perpendicular to the ground.
So, PQ || ST
In ΔPQR and ΔSTR,
$\angle PQR = \angle STR$ (Both are right angles)
$\angle PRQ = \angle SRT$ (common angle)
$\Delta PQR \sim \Delta STR$ (AA criterion for similarity)

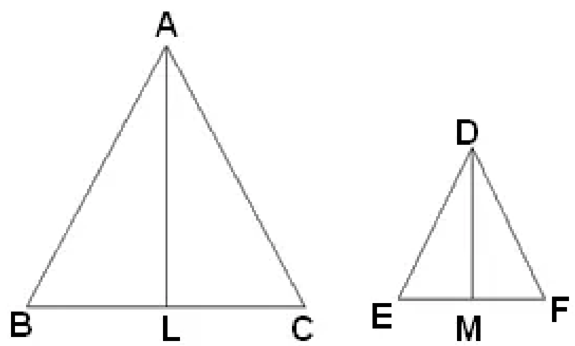
$\frac{PQ}{ST} = \frac{QR}{TR}$

$\Rightarrow \frac{h}{6} = \frac{12}{3}$

$\Rightarrow h = 24$ feet

Hence, the height of the pole is 24 feet.

Answer 6.



The ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding altitudes.

$\therefore \frac{area(\Delta ABC)}{area(\Delta PQR)} = \frac{AL^2}{DM^2}$

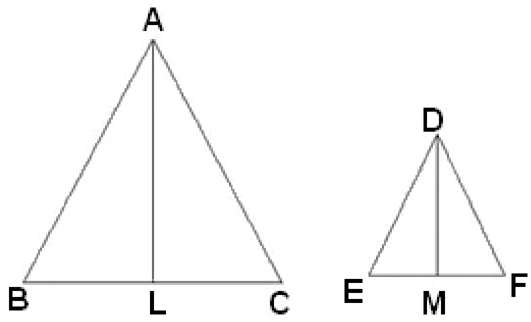
$\Rightarrow \frac{16}{9} = \frac{AL^2}{1.8^2}$

$\Rightarrow AL^2 = \frac{16 \times 3.24}{9}$

$\Rightarrow AL^2 = 5.76$

$\Rightarrow AL = 2.4cm$

Answer 7.



The ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding sides.

$$\therefore \frac{area(\Delta ABC)}{area(\Delta PQR)} = \frac{AB^2}{DE^2}$$

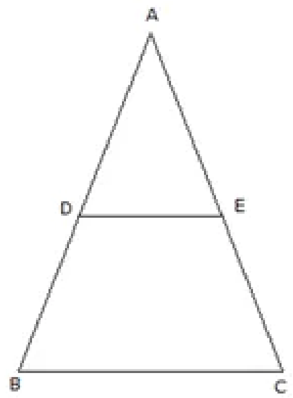
$$\Rightarrow \frac{169}{121} = \frac{26^2}{DE^2}$$

$$\Rightarrow DE^2 = \frac{121 \times 676}{169}$$

$$\Rightarrow DE^2 = 484$$

$$\Rightarrow DE = 22cm$$

Answer 8.



Area(Δ ADE) = area(trapezium BCED)

$\Rightarrow$ Area(Δ ADE) + Area(Δ ADE) = area(trapezium BCED) + Area(ΔADE)

$\Rightarrow$ 2 Area(Δ ADE) = Area(Δ ABC)

In Δ ADE and Δ ABC,

$\angle ADE = \angle B$ (corresponding angles)

$\angle A = \angle A$

Therefore, $\Delta ADE \sim \Delta ABC$

$$\therefore \frac{area(\Delta ADE)}{area(\Delta ABC)} = \frac{AD^2}{AB^2}$$

$$\Rightarrow \frac{area(\Delta ADE)}{2 \times area(\Delta ADE)} = \frac{AD^2}{AB^2}$$

$$\Rightarrow \frac{1}{2} = \left(\frac{AD}{AB}\right)^2$$

$$\Rightarrow \frac{AD}{AB} = \frac{1}{\sqrt{2}}$$

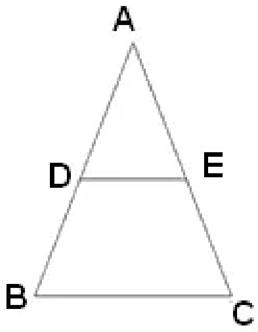
$$\Rightarrow AB = \sqrt{2}AD$$

$$\Rightarrow AB = \sqrt{2}(AB - BD)$$

$$\Rightarrow (\sqrt{2} - 1)AB = \sqrt{2}BD$$

$$\Rightarrow \frac{BD}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}} = \frac{2 - \sqrt{2}}{2}$$

Answer 9.



$$AD : DB = 2:3$$

$$AB = AD + DB = 2+3 = 5$$

(i) $$\frac{area(\Delta ADE)}{area(\Delta ABC)} = \frac{AD^2}{AB^2}$$

$$\Rightarrow \frac{area(\Delta ADE)}{area(\Delta ABC)} = \frac{2^2}{5^2}$$

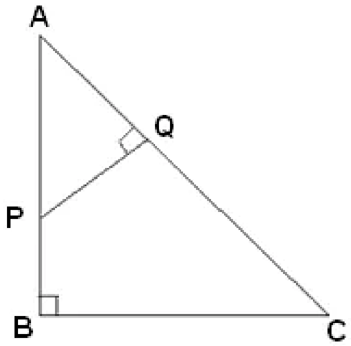
$$\Rightarrow \frac{area(\Delta ADE)}{area(\Delta ABC)} = \frac{4}{25}$$

(ii) $$\frac{area(trapezium EDBC)}{area(\Delta ABC)} = \frac{area(\Delta ABC) - area(\Delta ADE)}{area(\Delta ABC)}$$

$$\Rightarrow \frac{area(trapezium EDBC)}{area(\Delta ABC)} = \frac{25 - 4}{25}$$

$$\Rightarrow \frac{area(trapezium EDBC)}{area(\Delta ABC)} = \frac{21}{25}$$

Answer 10.



In ΔAQP and ΔABC

$\angle A = \angle A$

$\angle PQA = \angle ABC$ (right angles)

Therefore, $\Delta AQP \sim \Delta ABC$

(i) By Pythagoras theorem,

$$BC^2 = AC^2 - AB^2$$
$$\Rightarrow BC^2 = 10^2 - 8^2$$
$$\Rightarrow BC^2 = 100 - 64$$
$$\Rightarrow BC^2 = 36$$
$$\Rightarrow BC = 6cm$$

$$\text{Area } (\Delta ABC) = \frac{1}{2} \times AB \times BC$$

$$\text{Area } (\Delta ABC) = \frac{1}{2} \times 8 \times 6$$

$$\text{Area } (\Delta ABC) = 24cm^2$$

Since $\Delta AQP \sim \Delta ABC$

$$\frac{\text{Area}(\Delta AQP)}{\text{Area}(\Delta ABC)} = \frac{PQ^2}{BC^2}$$
$$\Rightarrow \frac{\text{Area}(\Delta AQP)}{24} = \frac{3^2}{6^2}$$
$$\Rightarrow \text{Area}(\Delta AQP) = \frac{9 \times 24}{36}$$
$$\Rightarrow \text{Area}(\Delta AQP) = 6cm^2$$

(ii) Area (trapezium EDBC) = Area (ΔABC) - Area (ΔAQP)

Area (trapezium EDBC) = 24 – 6 = 18 cm^2

$$\frac{\text{Area}(\text{Trapezium}EDBC)}{\text{Area}(\Delta ABC)} = \frac{18}{24}$$
$$\Rightarrow \frac{\text{Area}(\text{Trapezium}EDBC)}{\text{Area}(\Delta ABC)} = \frac{3}{4}$$

Area (trapezium EDBC): Area (ΔABC) =3:4

Answer 11.

(i) Image length = 6 cm, Actual length = 4 cm.

$$\text{Scale factor} = \frac{\text{Image length}}{\text{Actual length}} = \frac{6}{4}$$

Scale factor = 1.5

Since the scale factor > 1

$\Rightarrow$ Type of size transformation = enlargement

(ii) Actual length = 12 cm, Image length = 15 cm

$$\text{Scale factor} = \frac{\text{Image length}}{\text{Actual length}} = \frac{15}{12}$$

Scale factor = 1.25

Since the scale factor > 1

$\Rightarrow$ Type of size transformation = enlargement

(iii) Image length = 8 cm, Actual length = 20 cm.

$$\text{Scale factor} = \frac{\text{Image length}}{\text{Actual length}} = \frac{8}{20}$$

Scale factor = 0.4

Since the scale factor < 1 and >0

$\Rightarrow$ Type of size transformation = reduction

(iv) Actual area = $64 m^2$, Model area = $100 cm^2$

Actual area = $64 \times 10000\ cm^2 = 640000\ cm^2$

$$\text{Scale factor} = \sqrt{\frac{\text{Model Area}}{\text{Actual Area}}} = \sqrt{\frac{100}{640000}} = \sqrt{\frac{1}{6400}} = \frac{1}{80}$$

Scale factor = 0.0125

Since the scale factor < 1 and >0

$\Rightarrow$ Type of size transformation = reduction

(v) Model area = $75 cm^2$, Actual area = $3 m^2$

Actual area = $3 \times 10000\ cm^2 = 30000\ cm^2$

$$\text{Scale factor} = \sqrt{\frac{\text{Model Area}}{\text{Actual Area}}} = \sqrt{\frac{75}{30000}} = \sqrt{\frac{1}{400}} = \frac{1}{20}$$

Scale factor = 0.05

Since the scale factor < 1 and >0

$\Rightarrow$ Type of size transformation = reduction

(vi) Model volume = 200 cm^3, Actual volume = 8 m^3

Actual volume = 8 x 1000000 cm^2 = 8000000 cm^2

$$\text{Scale factor} = \sqrt{\frac{\text{Model Volume}}{\text{Actual Volume}}} = \sqrt{\frac{200}{8000000}} = \sqrt{\frac{1}{40000}} = \frac{1}{200}$$

Scale factor = 0.005

Since the scale factor < 1 and >0

$\Rightarrow$ Type of size transformation = reduction

Answer 12.

(i) $\frac{\text{Image length}}{\text{Actual length}} = \text{Scale factor}$

$$\frac{B'C'}{8} = 0.6$$
$$\Rightarrow B'C' = 8 \times 0.6$$
$$\Rightarrow B'C' = 4.8cm$$

(ii) $\frac{\text{Image length}}{\text{Actual length}} = \text{Scale factor}$

$$\frac{A'B'}{AB} = 0.6$$
$$\Rightarrow AB = \frac{5.4}{0.6}$$
$$\Rightarrow AB = 9cm$$

Answer 13.

(i) $\frac{\text{Image length}}{\text{Actual length}} = \text{Scale factor}$

$$\frac{A'B'}{AB} = 5$$
$$\Rightarrow A'B' = 4 \times 5$$
$$\Rightarrow A'B' = 20cm$$

(ii) $\frac{\text{Image length}}{\text{Actual length}} = \text{Scale factor}$

$$\frac{B'C'}{BC} = 5$$
$$\Rightarrow BC = \frac{16}{5}$$
$$\Rightarrow BC = 3.2cm$$

Answer 14.

$$\text{Scale factor} = \frac{\text{Image length}}{\text{Actual length}}$$

$$\text{Scale factor} = \frac{12}{8} = 1.5$$

$$\frac{X'Y'}{XY} = 1.5$$

$$\Rightarrow X'Y' = 1.5 \times 12$$

$$\Rightarrow X'Y' = 18\text{cm}$$

$$\frac{X'Z'}{XZ} = 1.5$$

$$\Rightarrow X'Z' = 1.5 \times 14$$

$$\Rightarrow X'Z' = 21\text{cm}$$

Answer 15.

Scale = 1:25000

(i) Actual length of AB = 3 x 250000 cm

$$= \frac{3 \times 250000}{100 \times 1000}\text{km}$$

$$= 7.5\text{km}$$

AB = 7.5 km

(ii) Actual length of BC = 4 x 250000 cm

$$= \frac{4 \times 250000}{100 \times 1000}\text{km}$$

$$= 10\text{km}$$

BC = 10 km

$$\text{Area}(\Delta ABC) = \frac{1}{2} \times AB \times BC$$

$$\text{Area}(\Delta ABC) = \frac{1}{2} \times 7.5 \times 10\text{km}^2$$

$$\text{Area}(\Delta ABC) = 37.5\text{km}^2$$

Area of plot = 37.5 km^2

Answer 16.

1.2m x 75 cm x 2 m = 1.2m x 0.75 m x 2 m

Scale factor = 1:20

Length = 2m

Actual length = 20 x length = 20 x 2 = 40m

Breadth = 0.75 m

Actual breadth = 20 x breadth = 20 x 0.75 = 15m

Height = 1.2 m

Actual height = 20 x height = 20 x 1.2 = 24.0m

Actual dimensions are = 24m x 15m x 40m

Answer 17.

Scale factor = 1:50000

(i) area of land represented on the map:

40 Sq km = 40 X $(100 \times 1000)^2$ [as 1 km = 100000 cm]

$= 40 \times 10^{10}$

$$\frac{\text{Area(map)}}{\text{Area(land)}} = \text{Scale}$$

$$\frac{\text{Area(map)}}{40 \times 10^{10}} = \frac{1}{(50000)^2}$$

$$\text{Area(map)} = \frac{40 \times 10^{10}}{(50000)^2} = \frac{4000}{25}$$

$$\text{Area(map)} = 160\text{cm}^2$$

(ii) 1 cm on the map = 50,000 cm on the land (as the scale is 1:50000)

1 km = 100000 cm = 2 X 50000 cm

$$\frac{\text{distance(map)}}{\text{distance(land)}} = \text{Scale}$$

$$\frac{1}{\text{distance(land)} \times (100000)} = \frac{1}{(50000)}$$

$$\text{Hence 1 cm on map} = \frac{50000}{100000}$$

= 0.5 km.

Answer 18.

(i) 1 cm on the map = 200,000 cm on the land (as the scale is 1:200000)

1 km = 100000 cm

$$\frac{\text{distance(map)}}{\text{distance(land)}} = \text{Scale}$$

$$\frac{2}{\text{distance(land)} \times (100000)} = \frac{1}{(200000)}$$

Hence 2 cm on map = $\frac{2 \times 200000}{100000}$

= 4 km.

(ii) 1 cm on the map = 200,000 cm on the land (as the scale is 1:200000)

1 cm^2 on the map = $(200000)^2$ on the land

1 km = 100000 cm ⇒ 1 km^2 = 100000 x 100000 cm^2

$$\frac{\text{distance(map)}}{\text{distance(land)}} = \text{Scale}$$

$$\frac{2}{\text{distance(land)} \times (100000)^2} = \frac{1}{(200000)^2}$$

Hence 2 cm^2 on map = $\frac{2 \times 200000 \times 200000}{100000 \times 100000}$

= 8 km^2.

(iii) area of land represented on the map:

20 Sq km = 20 X $(100 \times 1000)^2$ [as 1 km = 100000 cm]

= 20×10^{10}

$$\frac{\text{Area(map)}}{\text{Area(land)}} = \text{Scale}$$

$$\frac{\text{Area(map)}}{20 \times 10^{10}} = \frac{1}{(200000)^2}$$

$$\text{Area(map)} = \frac{20 \times 10^{10}}{(200000)^2} = \frac{20}{4}$$

$$\text{Area(map)} = 5cm^2$$

Answer 19.

Scale = 1:20000

(i) 1 cm on the map = 20000 cm on the land (as the scale is 1:20000)

1 km = 100000 cm

$$\frac{\text{distance(map)}}{\text{distance(land)}} = \text{Scale}$$

$$\frac{6}{\text{distance(land)} \times 100000} = \frac{1}{20000}$$

$$\text{Hence 6 cm on map} = \frac{6 \times 20000}{100000}$$

$$= 1.2 \text{ km.}$$

(ii) 1 km = 100000 cm

4 km = 400000 cm

$$\frac{\text{distance(map)}}{\text{distance(land)}} = \text{Scale}$$

$$\frac{\text{distance(map)}}{400000} = \frac{1}{20000}$$

$$\text{4 km distance on map} = \frac{400000}{20000}$$

$$= 20 \text{ cm}$$

(iii) area of lake represented on the map:

12 Sq km = 12 X $(100 \times 1000)^2$ [as 1 km = 100000 cm]

$= 12\text{X}10^{10}$

$$\frac{\text{Area(map)}}{\text{Area(land)}} = \text{Scale}$$

$$\frac{\text{Area(map)}}{12 \times 10^{10}} = \frac{1}{(20000)^2}$$

$$\text{Area(map)} = \frac{12 \times 10^{10}}{(20000)^2} = \frac{1200}{4}$$

$$\text{Area(map)} = 300\text{cm}^2$$

Answer 20.

Scale = 1:40

(i) The length of the model = 15 cm

The actual length = 15 x 40 = 600 cm = $\frac{600}{100}$ = 6m

(ii) Volume of the truck = 64 m^3

$$\frac{volume(model)}{volume(truck)} = Scale$$

$$\frac{volume(model)}{64 \times (100)^3} = \frac{1}{(40)^3}$$

$$Volume(model) = \frac{64000000}{64000}$$

$$Volume(model) = 1000cm^3$$

(iii) $$\frac{Area(model)}{Area(truck)} = Scale$$

$$\frac{30 \times (100)^2}{Area(truck)} = \frac{1}{(40)^2}$$

$$Area(truck) = 30 \times 1600 \times 10^4$$

$$Area(truck) = 4.8 \times 10^8 cm^2$$

Answer 21.

Scale = 1:500

(i) The length of the model = 1.2 m

The actual length = 1.2 x 500 = 600 m

(ii) $$\frac{Area(deck\ model)}{Area(deckship)} = Scale$$

$$\frac{1.6 \times 100 \times 100}{Area(deckship) \times 100 \times (1000)^2} = \frac{1}{(500)^2}$$

$$Area(deckship) = \frac{1.6 \times 2500}{10000}$$

$$Area(deckship) = 0.4 km^2$$

(iii) Volume of the ship = 1 km^3

$$\frac{volume(model)}{volume(ship)} = Scale$$

$$\frac{volume(model)}{1 \times (1000)^3} = \frac{1}{(500)^3}$$

$$Volume(model) = \frac{1000000000}{125000000}$$

$$Volume(model) = 8m^3$$

Answer 22.

scale = 1:25000

(i) In rectangle ABCD,

AB = 12 cm, BC = 16 cm

AC is the diagonal.

By Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$
$$AC^2 = 12^2 + 16^2$$
$$AC^2 = 144 + 256 = 400$$
$$\Rightarrow AC = 20cm$$

$$\because Scale = 1 : 25000$$
$$AC = 20 \times 25000cm$$
$$\Rightarrow AC = \frac{20 \times 25000}{100 \times 1000}km$$
$$\Rightarrow AC = 5km$$

(ii) Area ABCD = 12 x 16 x 25000 x 25000 cm^2

$$= \frac{12 \times 16 \times 25000 \times 25000}{100 \times 1000 \times 100 \times 1000}km^2$$
$$= \frac{120000}{10000}km^2$$
$$= 12km^2$$

Answer 23.

Scale = 1:25000

(i) Let AB = 225 cm and BC = 64 cm

Actual length of AB -

$$= \frac{225 \times 25000}{100 \times 1000} km$$

$$= \frac{5625}{100} km$$

$$= 56.25 km$$

Actual length of BC -

$$= \frac{64 \times 25000}{100 \times 1000} km$$

$$= \frac{16}{100} km$$

$$= 16 km$$

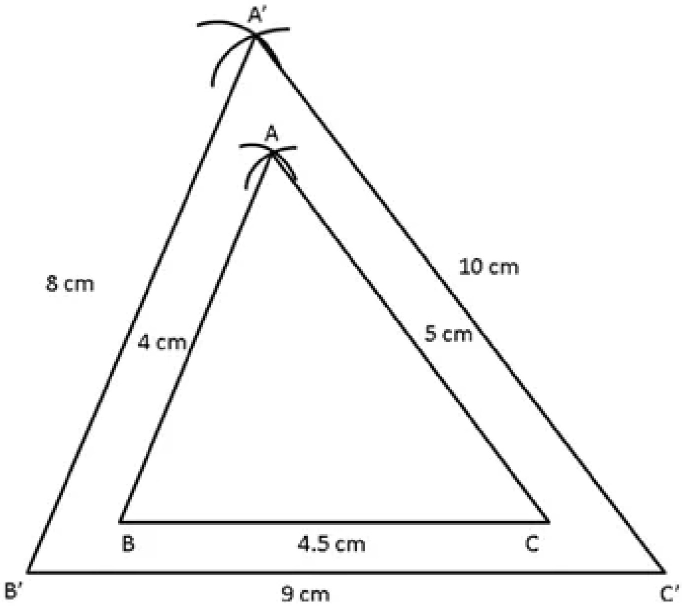
(ii) Area ABC $= \frac{1}{2} \times 12 \times 16 \times 25000 \times 25000 \ cm^2$

$$= \frac{1 \times 225 \times 64 \times 25000 \times 25000}{2 \times 100 \times 1000 \times 100 \times 1000} km^2$$

$$= \frac{9000000}{2 \times 10000} km^2$$

$$= 450 km^2$$

Answer 24.



Steps of Construction of the Image : :

1. Draw BC measuring 4 cm.
2. With B as the centre and radius 4.5 cm, make an arc above BC.
3. With C as the centre and radius 5 cm, to cut the previous arc at C.
4. ΔABC is the required triangle.

$$\text{Scale factor} = \frac{A'B'}{AB}$$

$$\Rightarrow 2 = \frac{A'B'}{4}$$

$\Rightarrow A'B' = 8$ cm

$$\text{Scale factor} = \frac{B'C'}{BC}$$

$$\Rightarrow 2 = \frac{B'C'}{4.5}$$

$\Rightarrow B'C' = 9$ cm

$$\text{Scale factor} = \frac{A'C'}{AC}$$

$$\Rightarrow 2 = \frac{A'C'}{5}$$

$\Rightarrow A'C' = 10$ cm

Steps of Construction of the Image : :

1. Draw B'C' measuring 9 cm.
2. With B' as the centre and radius 8 cm, make an arc above B'C'.
3. With C' as the centre and radius 9 cm, to cut the previous arc at C'.

4. $\Delta A'B'C'$ is the required image of the ΔABC.

On measuring the sides, we get

$$\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC} = \text{Scale factor} = 2$$